Roll No. | | | | | | | | | | |

ANNA UNIVERSITY (UNIVERSITY DEPARTMENTS)

B.E/ B.Tech/B.Arch (Full time)- END SEMESTER EXAMINATIONS (APRIL/MAY 2024)

Semester III

MA5302 - Discrete Mathematics

(Regulation 2019)

Time: 3 hrs — Max. Marks:100

CO1	Understand the validity of the logical arguments, mathematical proofs and correctness of the algorithm.
CO2	Apply combinatorial counting techniques in solving combinatorial related problems.
CO3	Use graph models and their connectivity, traversability in solving real world problems.
CO4	Understand the significance of algebraic structural ideas used in coding theory and cryptography.
CO5	Apply Boolean laws and Boolean functions in combinatorial circuit designs.

BL - Bloom's Taxonomy Levels

L1 - Remembering; L2 - Understanding; L3 - Applying; L4 - Analysing; L5 - Evaluating; L6 - Creating.

PART - A (10 × 2 = 20 Marks)

Answer ALL Questions.

Q.No	Questions	Marks	CO	BL
1.	Give an indirect proof of "If $3n + 2$ is an odd integer, then n is an odd integer".	2	1	L2
2.	Show that $\sim (P \wedge Q) \rightarrow (\sim P \vee (\sim P \vee Q)) \Rightarrow \sim P \vee Q$.	2	1	L2
3.	If 7 colours are used to paint 50 bicycles then show that at least 8 bicycles will be of same colour.	2	2	L2
4.	In how many ways can 9 people be seated in a circle?	2	2	L1
5.	For the degree sequence (3, 3, 3, 3, 2), does there exists a simple graph with 5 vertices? If so draw such graph.	2	3	L2
6.	How many vertices does a regular graph of degree 5 with 10 edges have?	2	3	L1
7.	Prove that every subgroup of an abelian group is a normal subgroup.	2	4	L2
8.	Find the identity element of the group of integers with the binary operation defined by $a * b = a + b + 2$ for all $a, b \in Z$.	2	4	L2
9.	Consider the set $D_{50} = \{1, 2, 5, 10, 25, 50\}$ and the relation | (divides) be a partial order relation on D_{50}. Draw the Hasse diagram for D_{50}.	2	5	L2
10.	In a complemented distributive lattice, if $a \leq b$ then prove that $a \wedge \overline{b} = 0$.	2	5	L2

PART - B (5 × 13 = 65 Marks)

(Restrict to a maximum of 2 subdivisions)

Q.No	Questions	Marks	CO	BL
11(a)	(i) Obtain the principal conjunctive normal form of $(\sim P \rightarrow R) \wedge (Q \rightleftarrows P)$. Hence find principal disjunctive normal form.	6	1	L3
	(ii) Prove that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)(P(x) \wedge (\exists x)Q(x)$.	7	1	L4
	(OR)			

11(b)	(i) Show that the following premises are inconsistent. (1) If Jack misses many classes due to illness, then he fails in high school. (2)If Jack fails in high school, then he is uneducated. (3) If Jack reads a lot of books, then he is not uneducated (4) Jack misses many classes due to illness and reads a lot of books.	6	1	L3
	(ii) Show that $((P \vee Q) \wedge \sim (\sim P \wedge (\sim Q \vee \sim R))) \vee (\sim P \wedge \sim Q) \vee (\sim P \wedge \sim R)$ is a tautology.	7	1	L4
12(a)	(i) Find the number of integers between 1 and 500 that are divisible by any of the integers 2, 3, 5 and 6.	7	2	L3
	(ii) How many integer solutions are there for $x+y+z=20$, subject to the constraints $x \geq -1, y \geq 0, z \geq 4$.	6	2	L4
	(OR)			
12(b)	(i)Use mathematical induction to prove that $(3^n + 7^n - 2)$ is divisible by 8, for $n \geq 1$.	6	2	L3
	(ii) Use the method of generating function to solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + 4^n; n \geq 2$ given that $a_0 = 2$ and $a_1 = 8$.	7	2	L4
13(a)	(i) Prove that a simple graph with n vertices must be connected if it has more than $\frac{(n-1)(n-2)}{2}$ edges.	6	3	L3
	(ii)If G is a connected simple graph with n vertices where $n \geq 3$, then prove that G is Hamiltonian if the degree of each vertex is at least $\frac{n}{2}$.	7	3	L4
	(OR)			
13(b)	(i) Prove that a connected multigraph has an Euler Tour if and only if each of its vertices has an even degree.	6	3	L3
	(ii) Prove that a connected graph G is bipartite if and only if all its cycles are of even length.	7	3	L4
14(a)	(i) State and prove Lagrange's theorem.	7	4	L3
	(ii)Prove that kernel of a homomorphism f from a group $\langle G, * \rangle$ to a group $\langle G', \Delta \rangle$ is a subgroup of $\langle G, * \rangle$.	6	4	L3
	(OR)			
14(b)	(i) State and prove Cayley's representation theorem.	7	4	L3
	(ii) Show that every group of order 3 is cyclic.	6	4	L3
	(OR)			
15(a)	(i) Show that isotone properties are true in lattice.	7	5	L3
	(ii) Prove that in a Boolean algebra complement of each element is unique.	6	5	L4
15(b)	(i) Let $(L, \leq)$ be a lattice in which $*$ and $\oplus$ denote the operations of meet and join respectively.Then prove that for any $a, b \in L$, $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$.	6	5	L3
	(ii) Prove that every chain is a distributive lattice.	7	5	L4

PART - C (1 × 15 = 15 Marks)

(Q. No. 16 is compulsory)

Q.No	Questions	Marks	CO	BL
16	(i) Prove that Demorgan's law are true in Boolean Algebra.	8	5	L3
	(ii) Establish the isomorphism of the following pair of graphs.	7	3	L4